Transport Properties of the Quark Gluon Plasma

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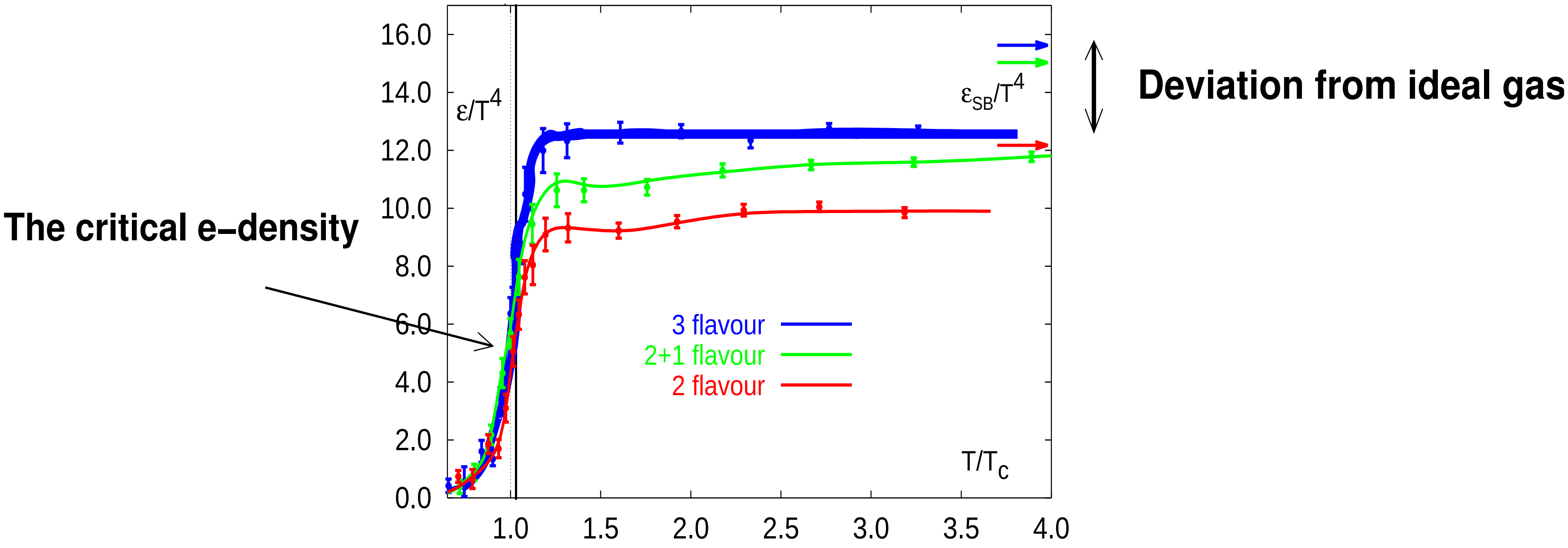
QCD is HOT: Lattice (RBRC-Bielefeld)



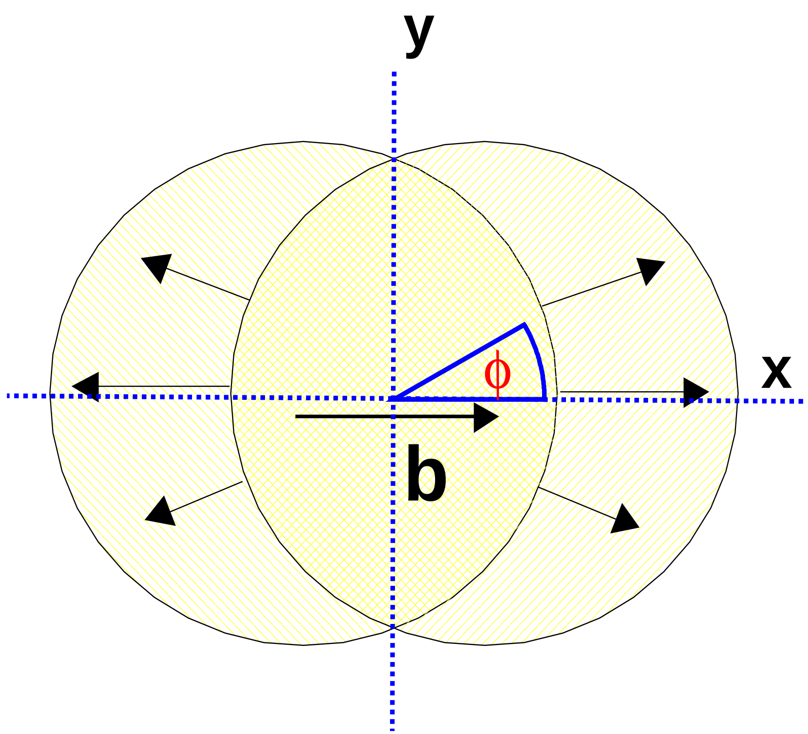
1. The critical energy density and temperature are

$$e_c \simeq 1\,\text{GeV}/\text{fm}^3 \qquad\qquad T_c \simeq 160\,\text{MeV}$$

2. What are its properties? Shear viscosity?

Need reach an energy density of e_c over a *Large* volume for *Long* enough.

Observation:



There is a large momentum anisotropy:

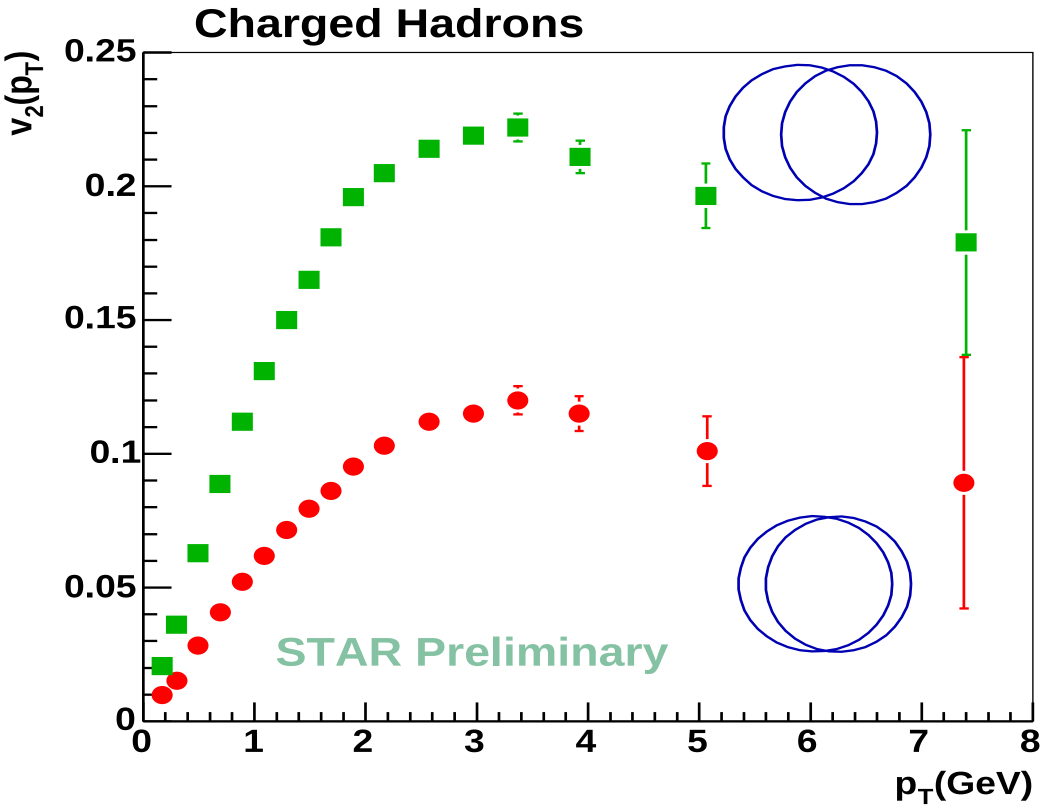
$$v_2 \equiv \frac{\left\langle p_x^2 - p_y^2 \right\rangle}{\left\langle p_x^2 + p_y^2 \right\rangle} \approx 20\%$$

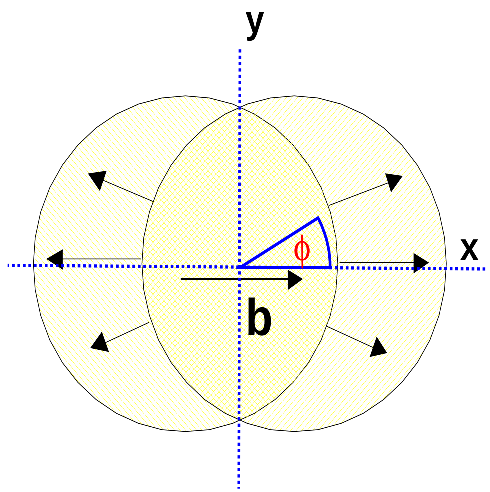
Interpretation

- The medium responds as a fluid to differences in X and Y pressure gradients

Data on Elliptic Flow:

$$\frac{1}{p_T}\frac{dN}{dp_T\,d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\,\left(1 + 2\,v_2(p_T)\cos(2\,\phi) + \ldots.\right)$$

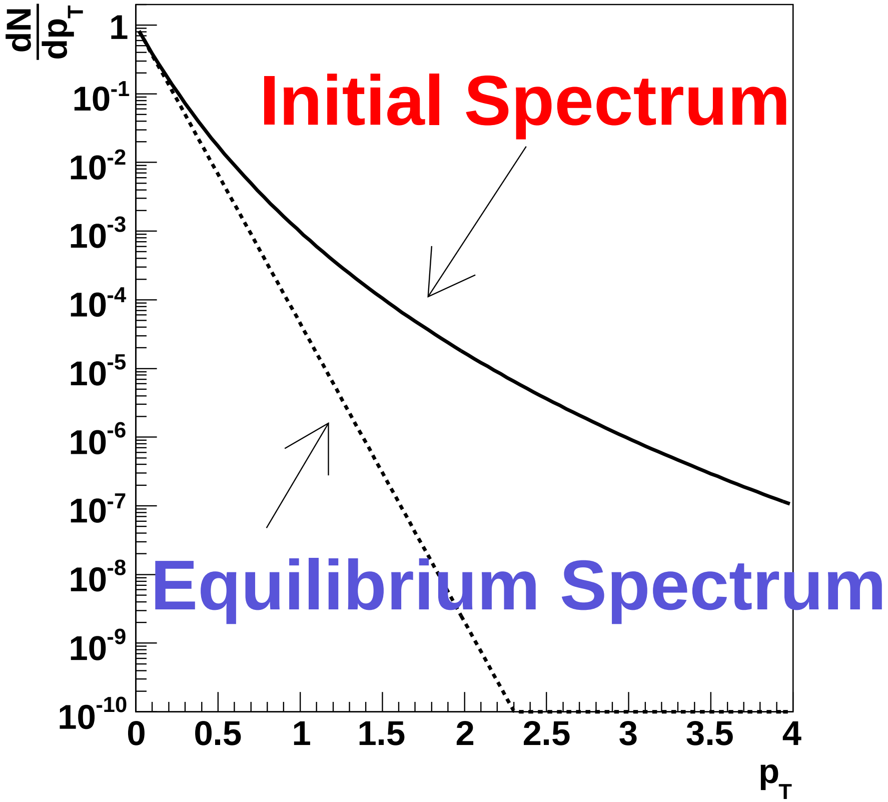




X:Y = $\left(1 + \underbrace{2v_2}_{\sim 0.4} : 1 - \underbrace{2v_2}_{\sim 0.4}\right)$

Elliptic flow is large X:Y $\sim 2.0 : 1$

Energy Loss of Fast Partons – Cartoon



- Power law initial spectrum:

$$\frac{dN}{dp_T} \propto \left(\frac{1}{p_T}\right)^{10}$$

- Exponential equilib. spectrum:

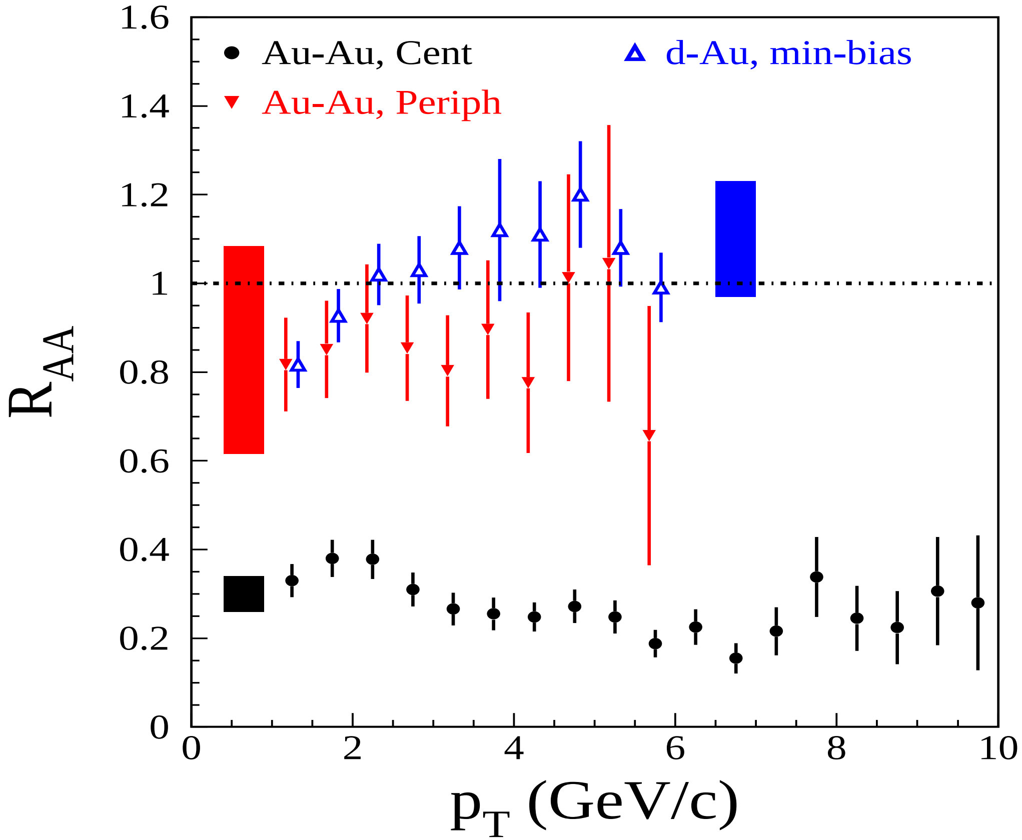
$$\frac{dN}{dp_T} \propto e^{-\frac{p_T}{T}}$$

The initial spectrum will lose energy and approach the equilibrium spectrum

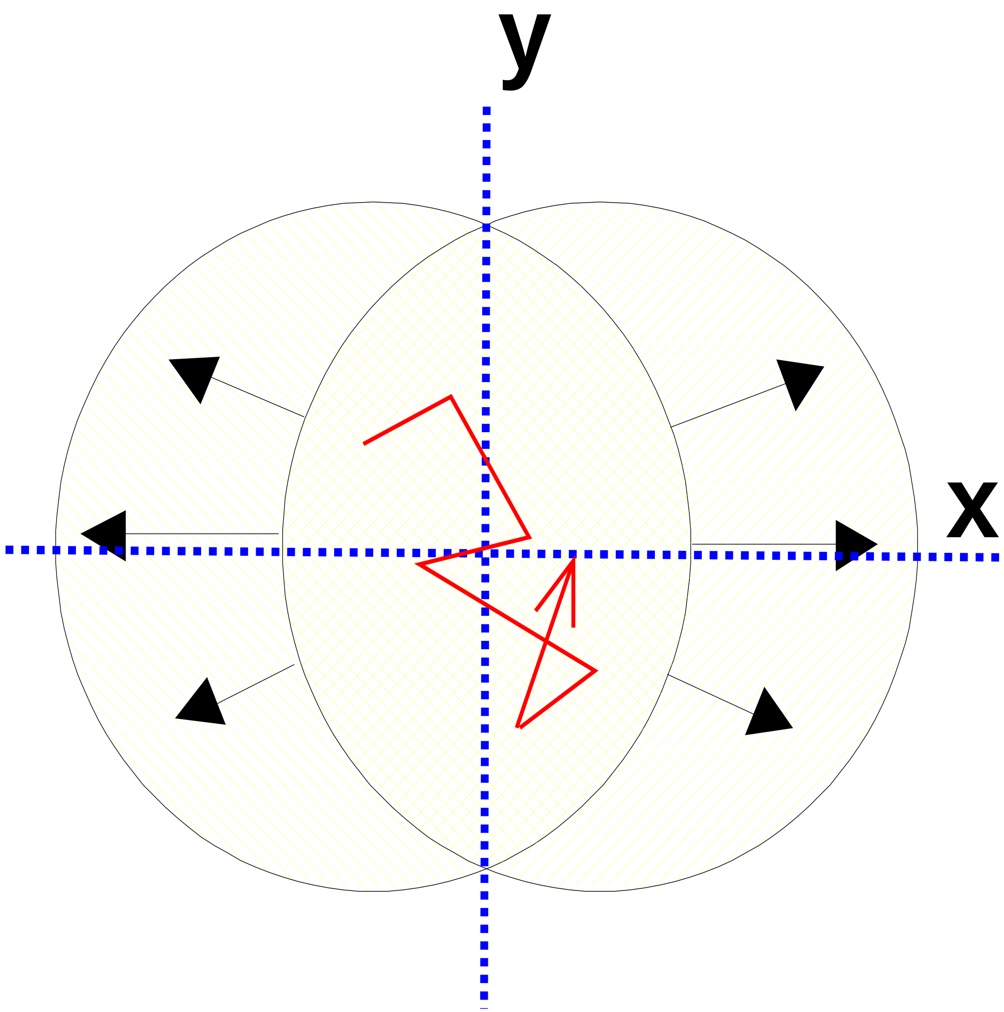
Tells something about density and interaction rates

Data on π^0 p_T spectrum

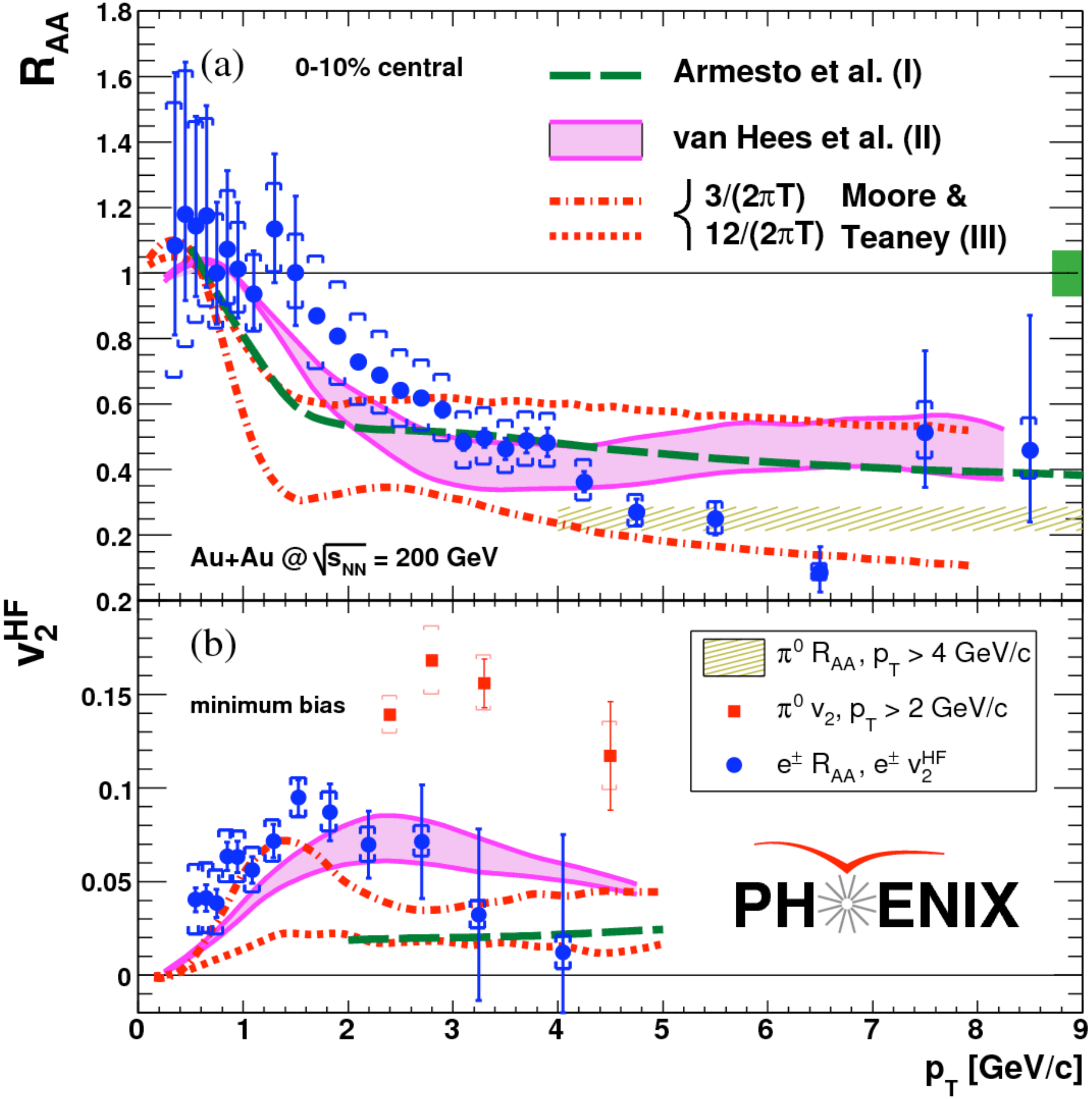
$$R_{AA} \equiv \frac{\left(\frac{dN}{p_T dp_T}\right)_{\text{In AuAu}}}{N_{\text{coll}}\left(\frac{dN}{p_T dp_T}\right)_{\text{In pp}}}$$



Heavy Quarks



The heavy quarks will either relax to the thermal spectrum and show the same v_2 as all thermal particles or not depending on the Drag/Diffusion coefficients and p_T.



R_{AA}
(a)
0-10% central
Armesto et al. (I)
van Hees et al. (II)
3/(2πT)
12/(2πT)
Moore & Teaney (III)
Au+Au @ $\sqrt{s_{NN}}$ = 200 GeV
v_2^{HF}
(b)
minimum bias
π^0 R_{AA}, p_T > 4 GeV/c
π^0 v_2, p_T > 2 GeV/c
$e^\pm$ R_{AA}, $e^\pm$ v_2^{HF}
PHENIX
p_T [GeV/c]

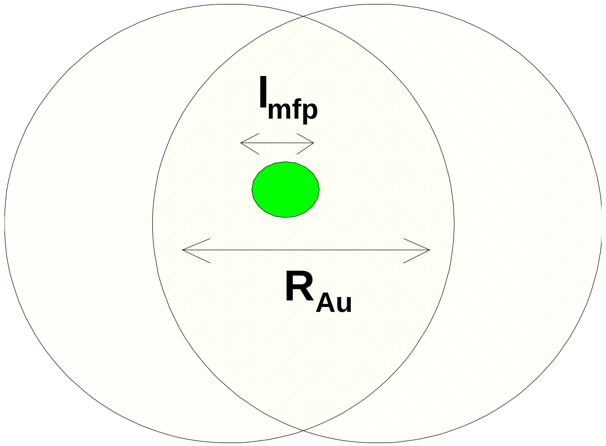
Data Recapitulation

1. Elliptic Flow – Soft event is strongly modified

2. Heavy Quarks – Suppressed and flowing

3. Energy loss significant

Hydrodynamics:



- For hydrodynamics need:

$$\frac{\ell_{\rm mfp}}{R_{\rm Au}} \ll 1$$

- How to define $\ell_{\rm mfp}$?

$$\ell_{\rm mfp} \sim \frac{\eta}{e+p} \qquad e+p = sT$$

Condition:

$$\underbrace{\frac{\eta}{s}}_{\text{Medium Property } \sim 1/\alpha_s^2} \times \underbrace{\frac{1}{RT}}_{\text{Experimental Property } \sim 1/2} \ll 1$$

Viscous Hydrodynamics Equations

$$T^{\mu\nu} = e u^\mu u^\nu + p g^{\mu\nu} + \pi^{\mu\nu}$$

- First order navier stokes theory

$$\pi = \pi^{\mu\nu}_{(1)} \equiv -\eta \left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \Delta^{\mu\nu} \nabla \cdot u \right)$$

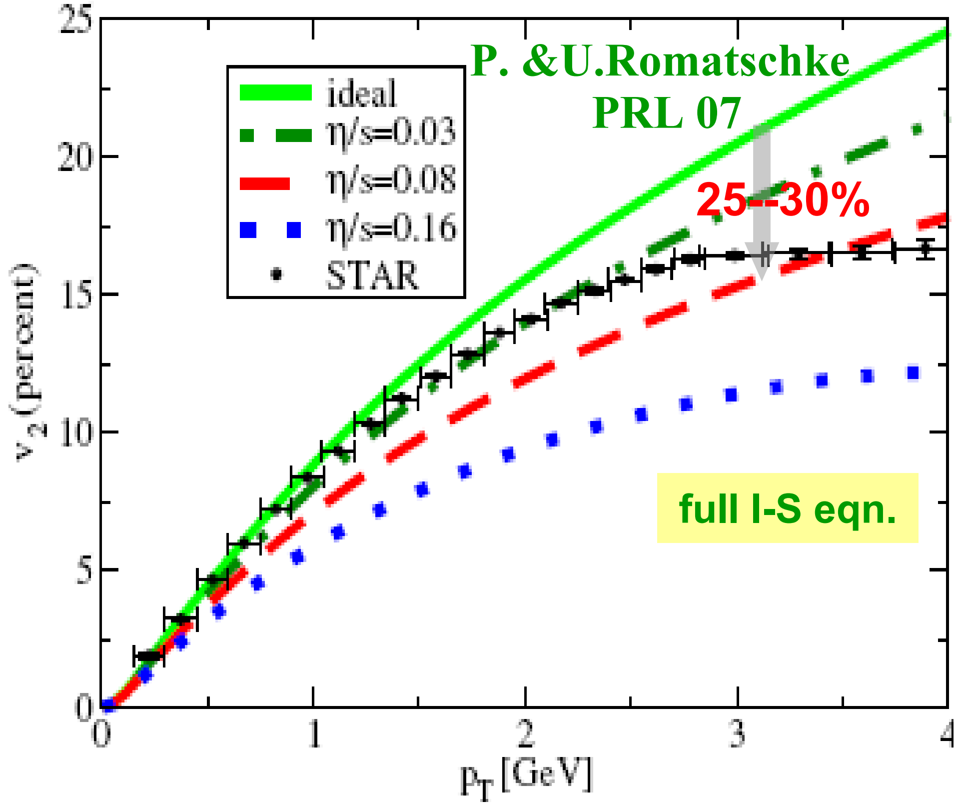
- Second order

$$\pi^{\mu\nu} = \overbrace{\pi^{\mu\nu}_{(1)}}^{O(\epsilon)} + \overbrace{\text{second derivatives}}^{O(\epsilon^2)}$$

- For example:

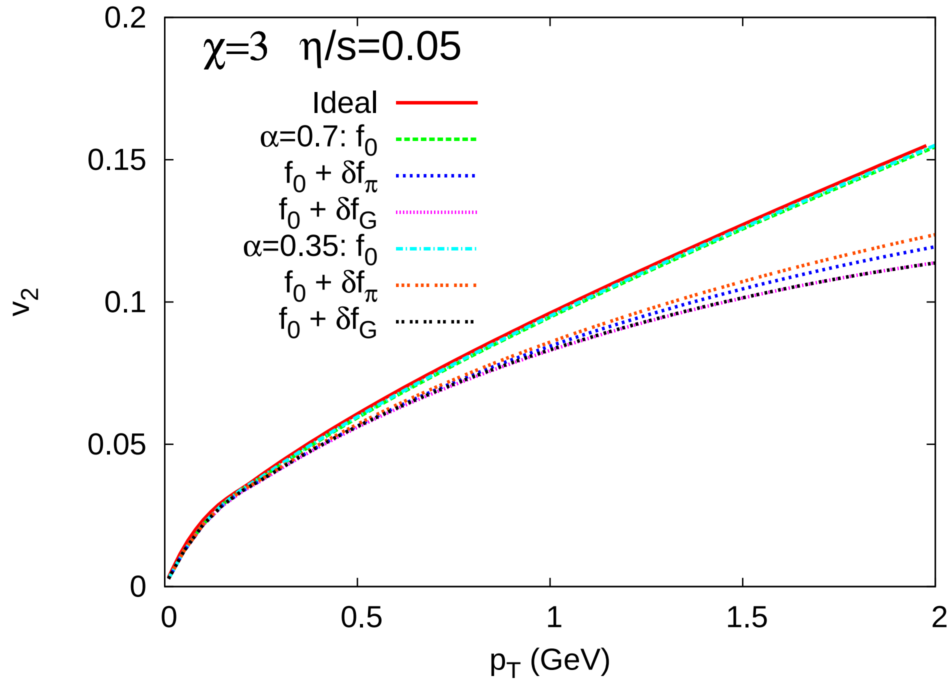
$$\pi^{\mu\nu} = \pi^{\mu\nu}_{(1)} - \tau_\pi D_t \pi^{\langle\mu\nu\rangle} + \text{Other 2nd derivs}$$

The best calculation so far: (Romatschke& Romatschke)



The elliptic flow can not be described unless $\frac{\eta}{s} < 0.4$

Independent of second derivative terms (DT and K. Dusling)



Gradient expansion is working. Temperature is a good concept.

Worse at larger viscosities and larger p_T

What does $\eta/s < 0.4$ mean theoretically?

- Perturbation theory: (Baym and Pethick. Arnold, Moore, Yaffe)
  - Kinetic theory of quarks and gluons + soft gauge fields + collinear emission

$$\frac{\eta}{s} \simeq 0.3 \left(\frac{0.5}{\alpha_s}\right)^2$$

- $\mathcal{N} = 4$ Super Yang Mills at strong coupling (Kovtun, Son, Starinets, Policastro)
  - No quasi-particles.

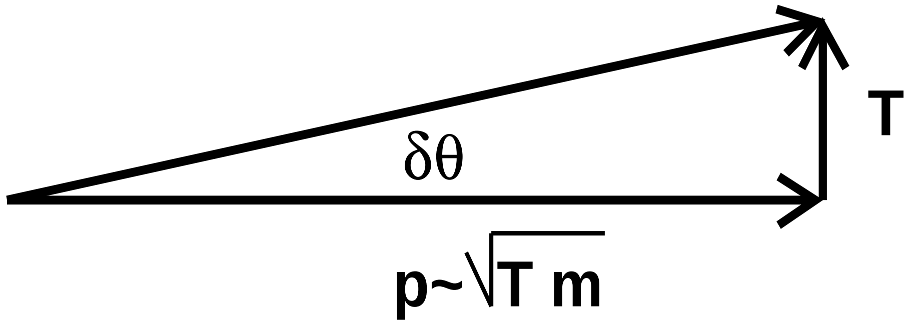
$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad \Longrightarrow \qquad \text{Conjectured Lower Bound}$$

The experimental results are within a factor of a few of the KSS bound

Heavy Quarks

Estimate of transport times with Heavy Quarks

- Put a heavy quark in this medium



$$\delta\theta \sim \sqrt{\frac{T}{m}}$$

- The charm quark undergoes a random walk suffering many collisions

- The relaxation time of the heavy quark is:

$$\tau_R^{\text{charm}} \sim \frac{M}{T}\tau_R^{\text{light}}$$

If you think you know the relaxation time you should be able to compute the charm spectrum.

Langevin description of heavy quark thermalization:

- Write down an equation of motion for the heavy quarks.

$$\frac{dx}{dt} = \frac{p}{M}$$
$$\frac{dp}{dt} = -\underbrace{\eta_D p}_{\text{Drag}} + \underbrace{\xi(t)}_{\text{Random Force}}$$

- The drag and the random force are related

$$\langle \xi_i(t)\xi_j(t')\rangle = \frac{\kappa}{3}\delta_{ij}\,\delta(t-t') \qquad \eta_D = \frac{\kappa}{2MT}$$

$\kappa =$ Mean Squared Momentum Transfer per Time

- Einstein related the diffusion coefficient to the mean squared momentum transfer

$$D = 2T^2/\kappa$$

All parameters are related to the heavy quark diffusion coefficient or κ

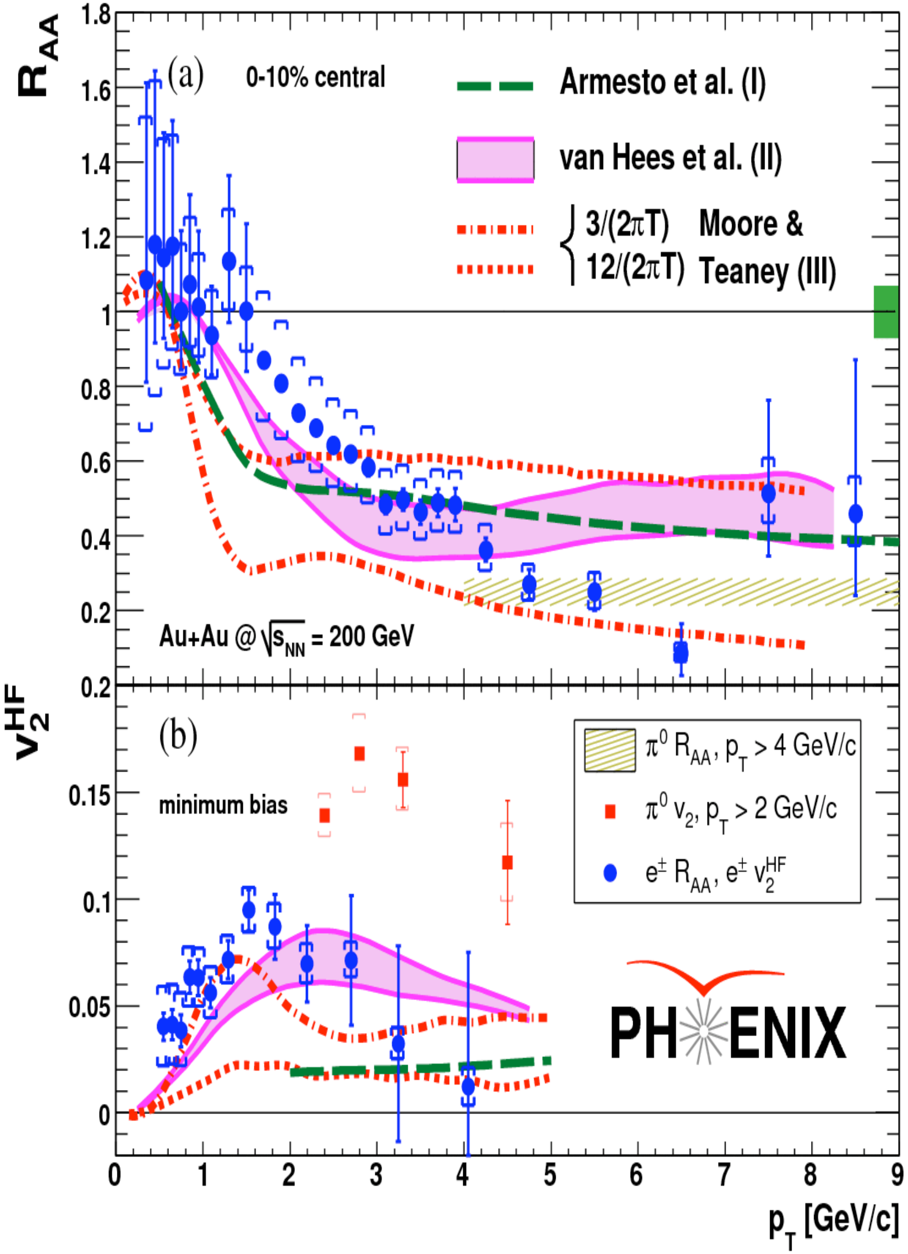
Application to Heavy Ion Collisions

- Generalize to mildly Relativistic quarks.
  - Assumes weak coupling.
  - Neglect radiative energy loss. The quark is not ultra-relativistic

$$\gamma v \lesssim \frac{1}{\alpha_s} \frac{m_{\rm D}}{T} \sim 6$$

  - Assumes a definite form for fluctuations

- Modeling
  - Input spectrum of charm and bottom quarks – from Cacciari *e.t. al*
  - Hadronize according to measured fragmentation functions
  - Electrons from charm and bottom semileptonic decays measured
  - Can not separated the charm and bottom contributions



Summary

1. Suppression and Elliptic Flow are intimately related.

2. From the suppression pattern, we estimate that

$$D \lesssim \frac{12}{2\pi T}$$

Hard to reproduce the elliptic flow and suppression at the same time.

Computing heavy quark diffusion coefficient

- Compute at weak coupling $\longrightarrow$ Kinetic Theory
- Lattice $\longrightarrow$ Hard
- Compute at strong coupling $\longrightarrow$ Model theories – AdS/CFT

Extrapolate to reality.

Giving the diffusion coefficient a rigorous definition

- Heavy Quarks are Quasi Classical

$$\lambda_{\text{de Broglie}} \sim \frac{\hbar}{\sqrt{MT}} \ll \frac{\hbar}{T}$$

- Compare the Langevin process to the microscopic theory

Langevin	Microscopic Theory
$\dfrac{dp}{dt} = -\eta_D p + \xi(t)$	$\dfrac{dp}{dt} = \mathcal{F}(t, \mathbf{x}) = qE(t, \mathbf{x})$

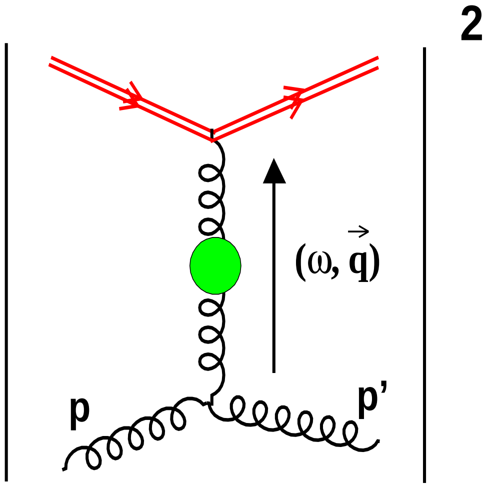
- Match the Langevin to the Microscopic Theory

Langevin	Microscopic Theory
$\kappa = \displaystyle\int dt \, \langle \xi(t) \, \xi(0) \rangle$	$\kappa = \displaystyle\int dt \, \langle \mathcal{F}(t, \mathbf{x}) \mathcal{F}(0, \mathbf{x}) \rangle_{HQ}$

Diffusion Coefficient $\leftrightarrow$ Electric Field Correlator

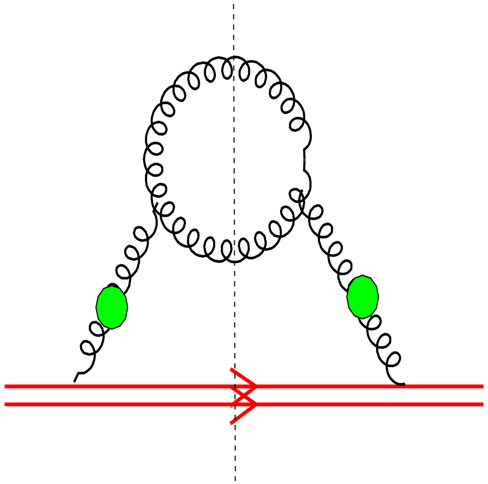
Computing κ – Kinetic Theory vs. Correlators

- κ is the mean squared momentum transfer per unit time:



$$\Longrightarrow \qquad \kappa = \int_{\mathbf{p},\mathbf{q}} \mathbf{q}^2 \;\; n(p)(1+n(p')) \;\; \left|M_{\text{glue}}\right|^2$$

- κ is an Chromo-electric field correlator (+ Wilson Lines):

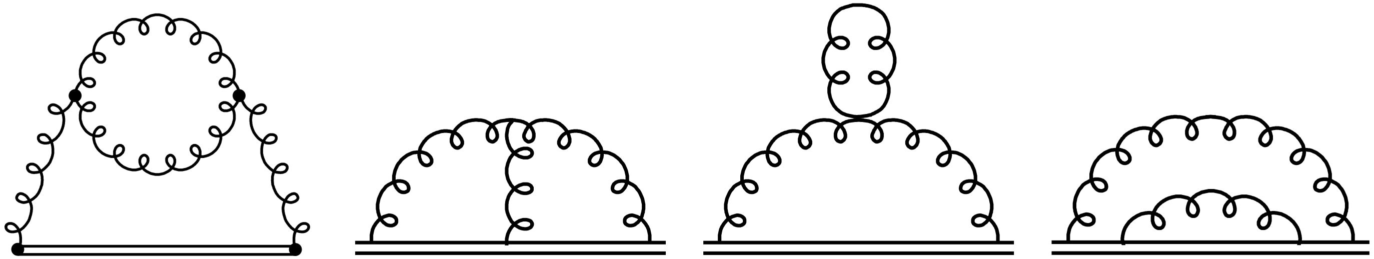


$$\Longrightarrow$$

The Same Thing

Beyond leading order (Guy D. Moore and Simon-Caron Huot)

(only transport coefficient known at NLO)



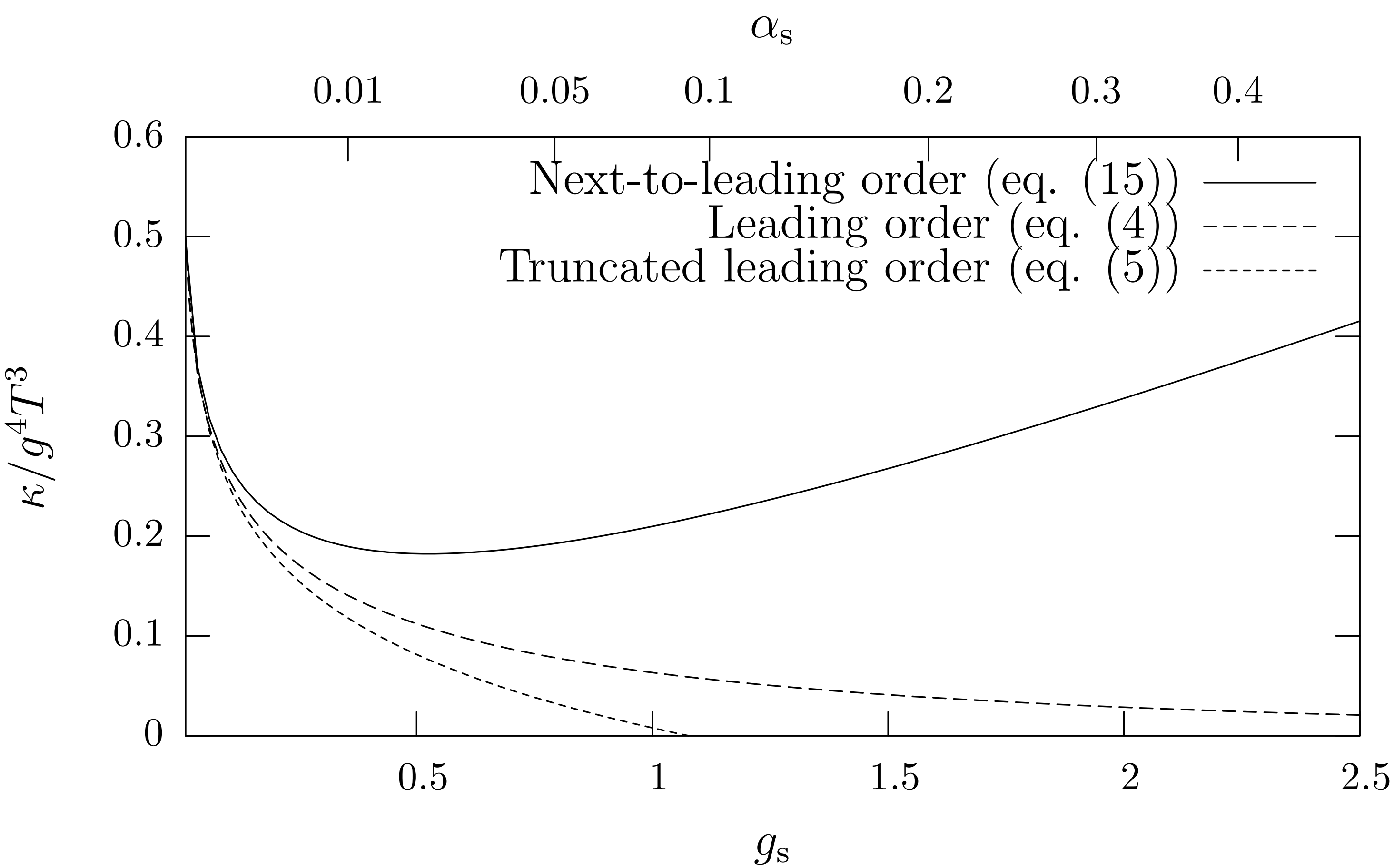
1. Perturbation theory in:

$$g_s \sim \frac{m_D}{T} \qquad \text{NOT} \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

2. Schematically:

$$\underbrace{\kappa}_{\text{diffusion rate}} = (g^4 T^3)\Big[\underbrace{C_0 \log\left(\frac{T}{m_D}\right) + C_1}_{\text{leading order}} + \underbrace{C_2 \frac{m_D}{T}}_{\text{NLO}}\Big]$$

(Guy D. Moore and Simon Carot-Huot)

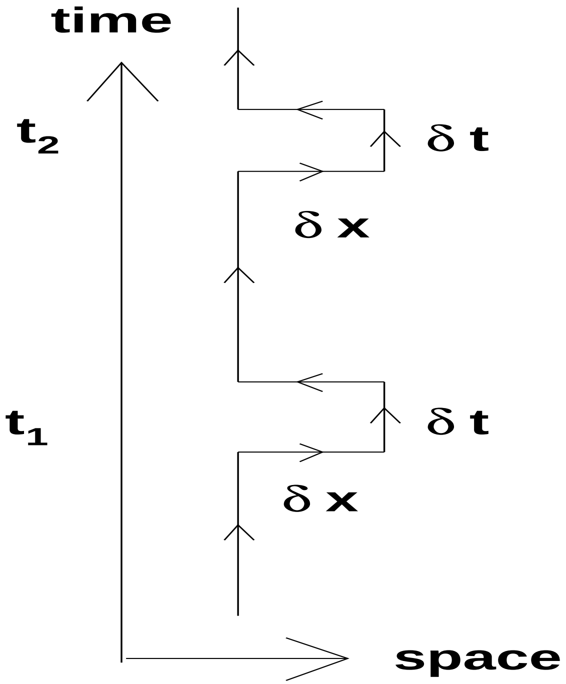


Perturbation theory fails for kinetics even for $T = M_Z$. More Resummation?

Computation in $\mathcal{N} = 4$ Super Yang Mills

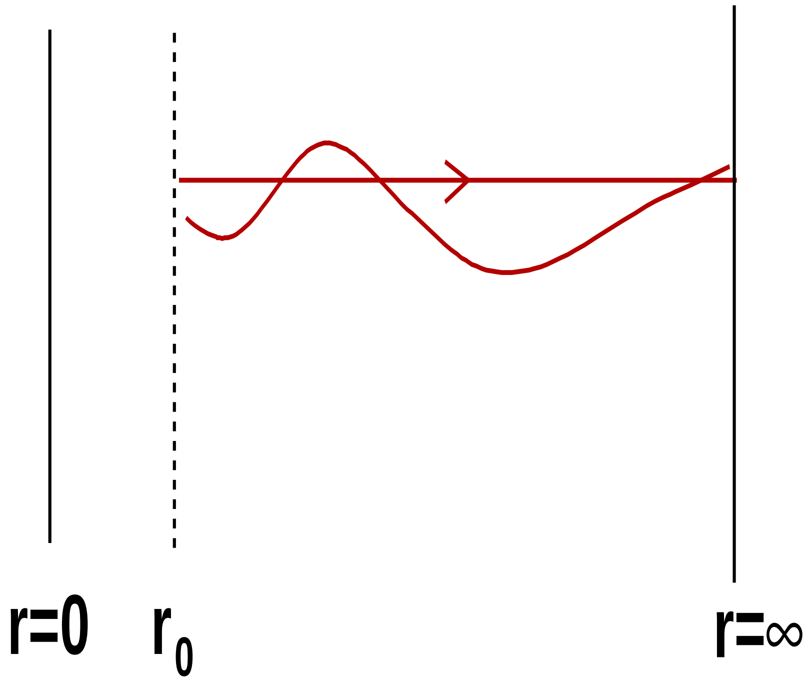
(DT, J. Casalderrey; Herzog et al)

Boundary



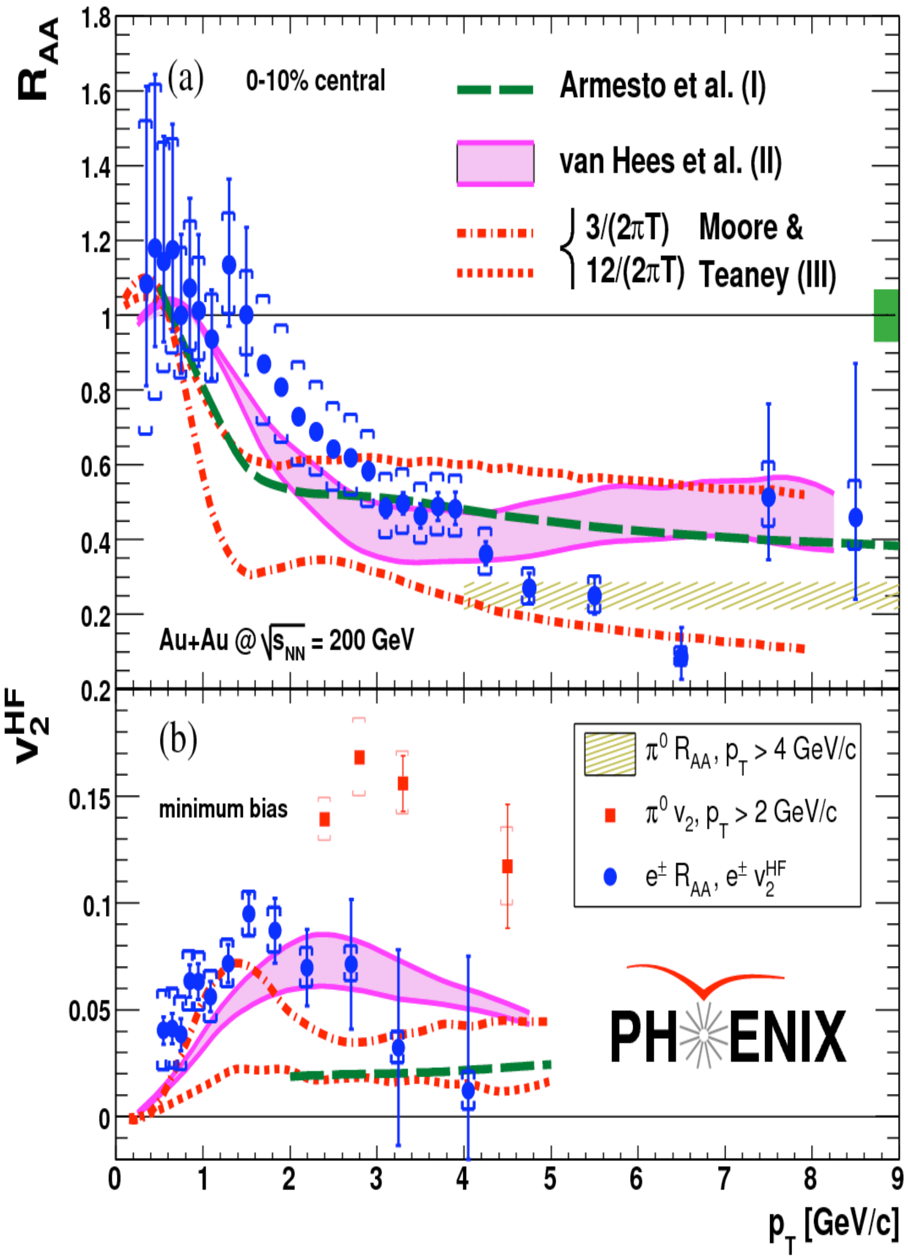
$\Longrightarrow$

Bulk



$W[\delta x]$ $\qquad\qquad$ $S_{\rm cl}[\delta x]$

$$\kappa = \sqrt{\lambda}\pi T^3 \qquad\qquad \lambda = g^2 N$$



Summary

- Perturbative QCD Estimates

$$D \approx \frac{2 \leftrightarrow 6}{2\pi T}$$

- Best guess for QCD from strong coupling

$$D \approx \frac{4.0 \leftrightarrow 8.0}{2\pi T}$$

Conclusions

1. Three important data:
   - Elliptic Flow – the soft event.
   - Suppression of high p_T particles.
   - Flow of heavy quarks
2. Data on flow and heavy quarks difficult to reconcile with kinetic theory
   - Quark and gluon quasi-particles not a good concept?
3. LHC: Much to learn about parton showers etc from this group
   - Next talk